A Maxwell-egyenletek és az elektromágneses hullám

Eltolási áram

Ampere:

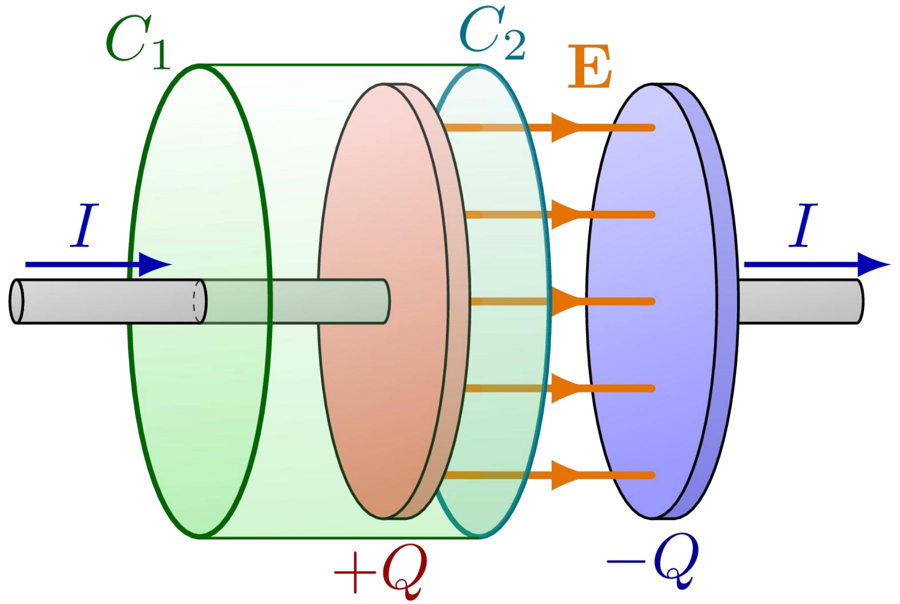
$$\oint \mathbf{B}\, d\mathbf{l} = \mu_0\, I$$

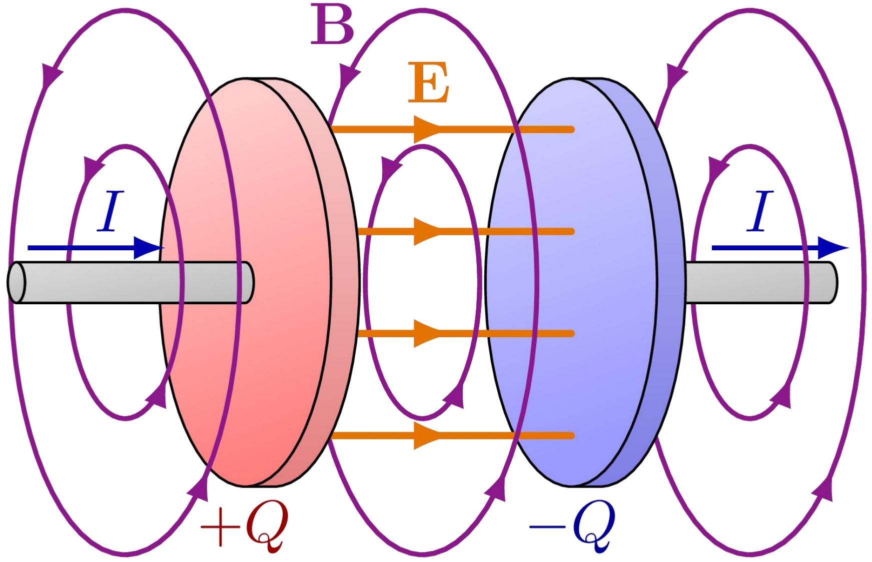
Ampere-Maxwell:

$$\oint \mathbf{B}\, d\mathbf{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{d\mathbf{E}}{dt} \right)$$

Anyagban: $$\nabla \times \mathbf{H} = \mathbf{j} + \frac{d\mathbf{H}}{dt}$$





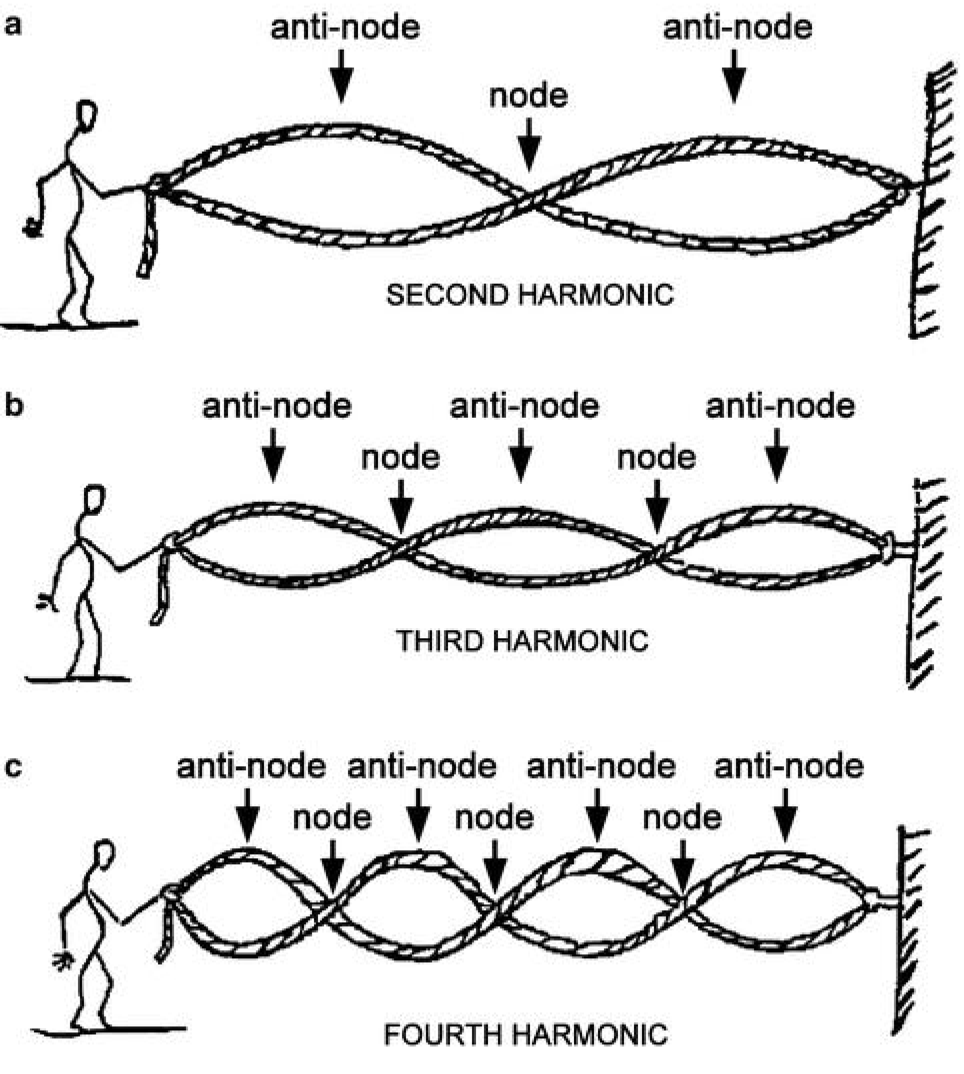
Maxwell-egyenletek vákuumban

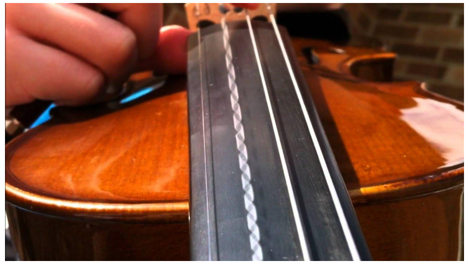
	Integrális alak (makroszkopikus)	Differenciális alak (makroszkopikus)
I. Elektromos Gauss-törvény	$\oiint \mathbf{E}\,d\mathbf{A} = \frac{q}{\varepsilon_0}$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
II. Mágneses Gauss-törvény	$\oiint \mathbf{B}\,d\mathbf{A} = 0$	$\nabla \cdot \mathbf{B} = 0$
III. Faraday-törvény	$\oint \mathbf{E}\,d\mathbf{l} = -\frac{d\,\phi_B}{dt}$	$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$
IV. Ampere-Maxwell-törvény	$\oint \mathbf{B}\,d\mathbf{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\,\phi_E}{dt} \right)$	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{d\mathbf{E}}{dt} \right)$

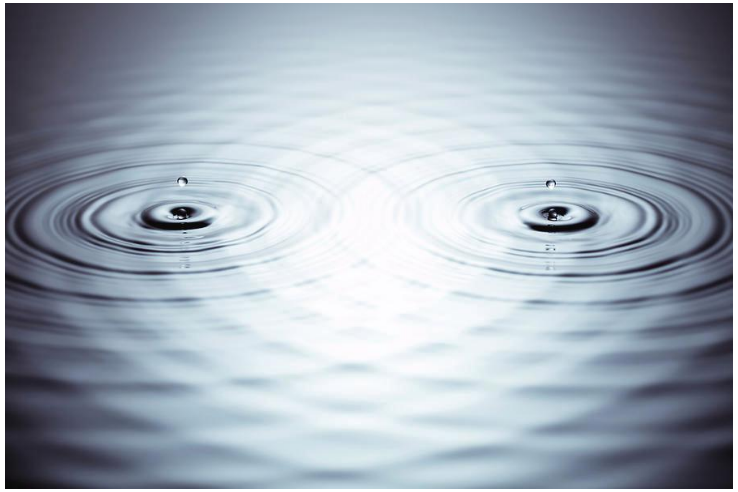
Maxwell-egyenletek anyag jelenlétében

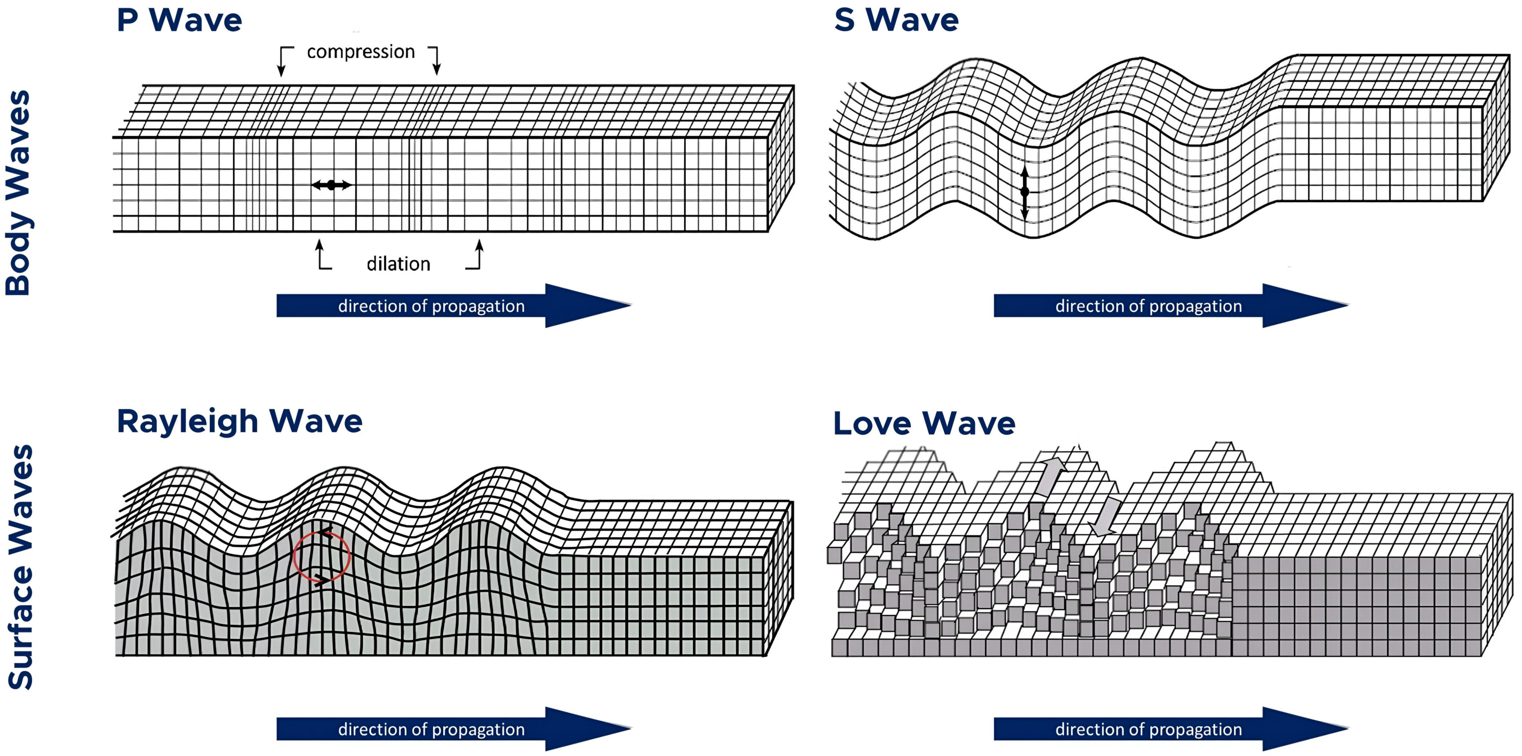
$q = q_{szabad} + q_{kötött}$ $I = I_{vezetési} + I_{eltolási}$	Integrális alak (makroszkopikus)	Differenciális alak (makroszkopikus)
I. Elektromos Gauss-törvény	$\oiint \mathbf{D}\, d\mathbf{A} = q_{sz}$	$\nabla \cdot \mathbf{D} = \rho_{sz}$
II. Mágneses Gauss-törvény	$\oiint \mathbf{H}\, d\mathbf{A} = 0$	$\nabla \cdot \mathbf{H} = 0$
III. Faraday-törvény	$\oint \mathbf{E}\, d\mathbf{l} = -\frac{d\,\phi_B}{dt}$	$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$
IV. Ampere-Maxwell-törvény	$\oint \mathbf{H}\, d\mathbf{l} = I_v + \varepsilon_0 \frac{d\,\phi_E}{dt}$	$\nabla \times \mathbf{H} = \mathbf{j}_v + \frac{d\mathbf{D}}{dt}$

Mechanikai hullámok









Hullámegyenlet

1D:

$$u(x,t) = A \cdot \sin(\omega t - kx)$$

$$\frac{dx}{dt} = \frac{\omega}{k} = v_{fázis} = c$$

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{\omega}{k}\right)^2 \cdot \frac{\partial^2 u}{\partial x^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$$

3D:

$$u\big(\mathbf{r}(x,y,z),t\big) = A \cdot \sin(\omega t - \mathbf{kr})$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = c^2 \cdot \nabla^2 u$$

Fázissebesség

2D hullám megoldás

2D hullám megoldás határfeltételekkel